STAT 400 UIUC

Answers for 7.1, 7.4

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1. Suppose the lifetime of a particular brand of light bulbs is normally distributed with standard deviation of $\sigma = 75$ hours and unknown mean.

a) What is the probability that in a random sample of 49 bulbs, the average lifetime $\overline{X}$ is within 21 hours of the overall average lifetime?

$\sigma = 75$, $\qquad n = 49$.

$$P(\mu - 21 < \overline{X} < \mu + 21) = P\left(\frac{(\mu - 21) - \mu}{75/\sqrt{49}} < Z < \frac{(\mu + 21) - \mu}{75/\sqrt{49}}\right)$$

$$= P(-1.96 < Z < 1.96) = \mathbf{0.95}.$$

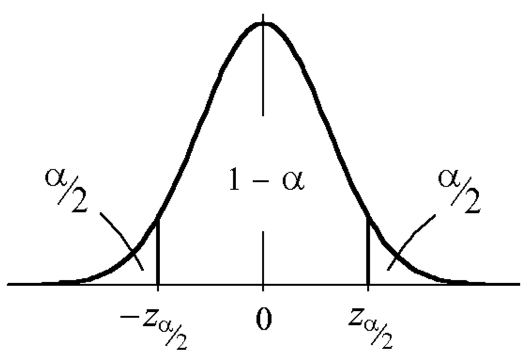
b) Suppose the sample average lifetime of the 49 bulbs is $\overline{x} = 843$ hours. Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

$(\overline{X} - 21, \overline{X} + 21)$ $\qquad$ $(\mathbf{822}, \mathbf{864})$

A **confidence interval** is a *range of numbers* believed to include an unknown population parameter. Associated with the interval is a measure of the *confidence* we have that the interval does indeed contain the parameter of interest.

A $(1 - \alpha)\,100\%$ confidence interval for the population mean μ when σ is known and sampling is done from a normal population, or with a large sample, is

$$\left(\overline{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\ ,\ \overline{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$



$$\overline{X} \qquad \pm \qquad z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

estimate (point estimate) — margin of error ε

$$\overline{X} \pm \varepsilon$$

$$\varepsilon = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

1. (continued)

Suppose the sample average lifetime of the 49 bulbs is $\bar{x} = 843$ hours.

b) Construct a 95% confidence interval for the overall average lifetime for light bulbs of this brand.

$\sigma = 75$ is known. $n = 49$ – large. The confidence interval : $\overline{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.

95% confidence level, $\alpha = 0.05$, $\alpha/2 = 0.025$, $z_{\alpha/2} = 1.96$.

$843 \pm 1.96 \cdot \frac{75}{\sqrt{49}}$ **843 ± 21** $\mathbf{(822, 864)}$

c) Construct a 90% confidence interval for the overall average lifetime for light bulbs.

90% confidence level, $\alpha = 0.10$, $\alpha/2 = 0.05$, $z_{\alpha/2} = 1.645$.

$843 \pm 1.645 \cdot \frac{75}{\sqrt{49}}$ **843 ± 17.625** $\mathbf{(825.375, 860.625)}$

d) Construct a 92% confidence interval for the overall average lifetime for light bulbs.

92% confidence level, $\alpha = 0.08$, $\alpha/2 = 0.04$, $z_{\alpha/2} = 1.75$.

$843 \pm 1.75 \cdot \frac{75}{\sqrt{49}}$ **843 ± 18.75** $\mathbf{(824.25, 861.75)}$

Minimum required sample size in estimating the population mean μ to within ε with $(1-\alpha)\,100\%$ confidence is

$$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2 .$$

Always round n up.

2. How many test runs of an automobile are required for determining its average miles-per-gallon rating on the highway to within 0.5 miles per gallon with 95% confidence, if a guess is that the variance of the population of miles per gallon is about 6.25?

$\varepsilon = 0.5$, $\sigma^2 = 6.25$, $\sigma = 2.5$,

95% confidence level, $\alpha = 0.05$, $\alpha/2 = 0.025$, $z_{\alpha/2} = 1.960$.

$$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2 = \left[\frac{1.96 \cdot 2.5}{0.5} \right]^2 = 96.04.$$

Round up. $n =$ **97**.

1. (continued)

e) What is the minimum sample size required if we wish to estimate the overall average lifetime for light bulbs to within 10 hours with 90% confidence?

$\varepsilon = 10$, $\sigma = 75$,

90% confidence level, $\alpha = 0.10$, $\alpha/2 = 0.05$, $z_{\alpha/2} = 1.645$.

$$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon} \right]^2 = \left[\frac{1.645 \cdot 75}{10} \right]^2 = 152.21390625.$$

Round up. $n =$ **153**.