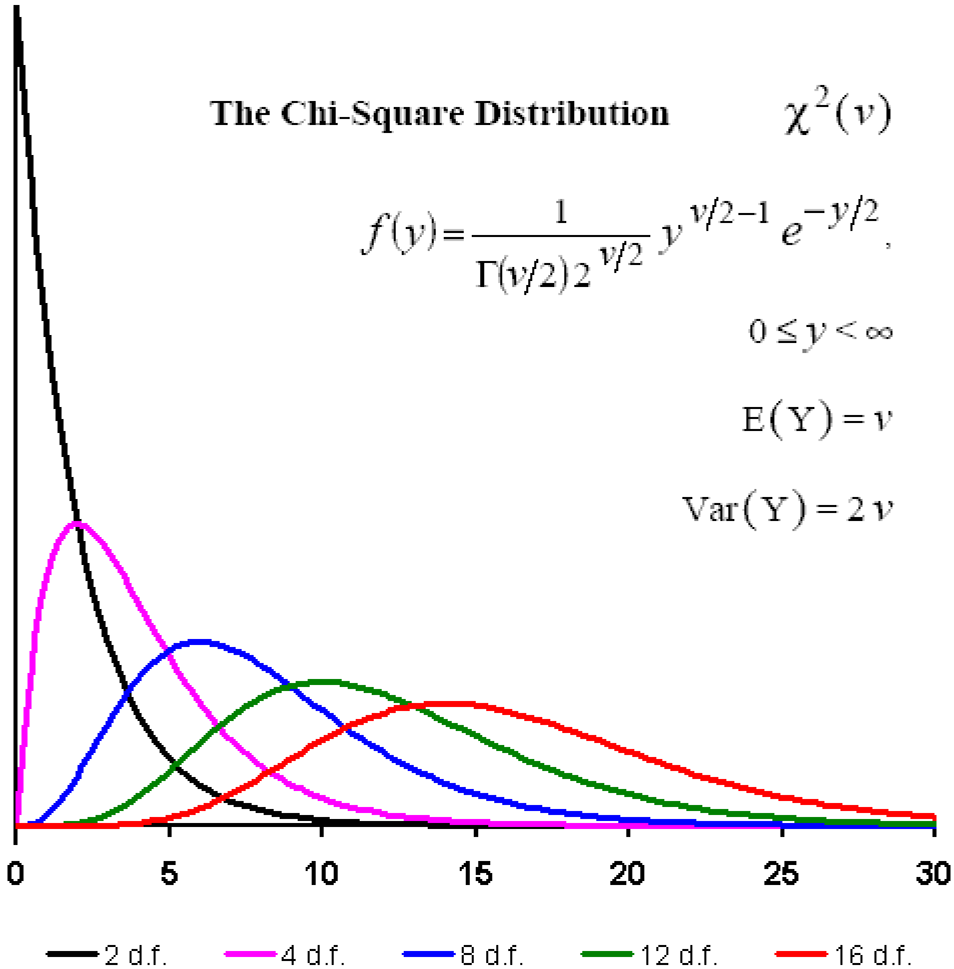
STAT 400
UIUC

Stepanov
Dalpiaz

Examples for 7.1 (Part 3)



EXCEL:

$= \text{CHIINV}(\alpha, \nu)$ gives $\chi^2_\alpha(\nu)$ for χ^2 distribution with ν degrees of freedom

$= \text{CHIDIST}(y, \nu)$ gives the upper tail probability for χ^2 distribution with ν degrees of freedom, $P(Y > y)$.

Recall:

If $X_1, X_2, \ldots, X_n$ are i.i.d. $\mathbf{N}(\mu, \sigma^2)$. Then

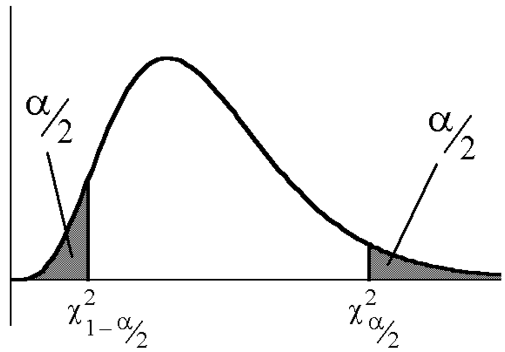
$$\frac{(n-1)\cdot S^2}{\sigma^2} = \frac{\sum (X_i - \overline{X})^2}{\sigma^2} \text{ is } \chi^2(n-1).$$

A $(1-\alpha)\,100\%$ confidence interval for the population variance σ^2

(where the population is assumed normal)

$$\left(\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}},\ \frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}} \right)$$

$n-1$ degrees of freedom



A $(1-\alpha)\,100\%$ confidence interval for the population standard deviation σ (where the population is assumed normal)

$$\left(\sqrt{\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}}},\ \sqrt{\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}}} \right) \quad \text{OR} \quad \left(s\cdot\sqrt{\frac{(n-1)}{\chi^2_{\alpha/2}}},\ s\cdot\sqrt{\frac{(n-1)}{\chi^2_{1-\alpha/2}}} \right)$$

$n-1$ degrees of freedom

1. A machine makes ½-inch ball bearings. In a random sample of 41 bearings, the sample standard deviation of the diameters of the bearings was 0.02 inch. Assume that the diameters of the bearings are approximately normally distributed. Construct a 90% confidence interval for the standard deviation of the diameters of the bearings.

2. The following random sample was obtained from $N(\mu, \sigma^2)$ distribution:

16 12 18 13 21 15 8 17

Recall: $\bar{x} = 15$, $s^2 = 16$, $s = 4$.

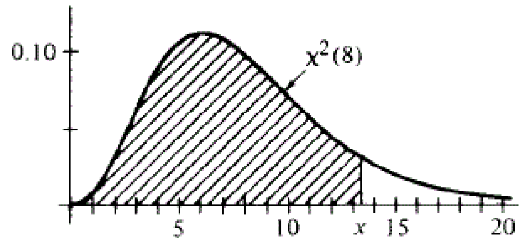
a) Construct a 95% confidence interval for the overall standard deviation.

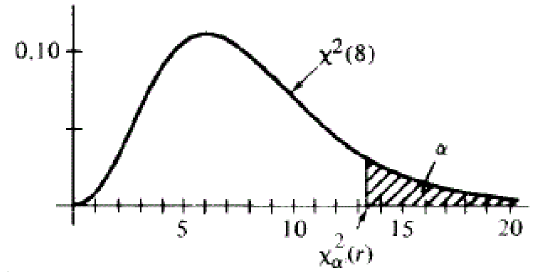
b) Construct a 95% confidence lower bound for the overall standard deviation.

c) Construct a 95% confidence upper bound for the overall standard deviation.

Table IV

The Chi-Square Distribution





$$P(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2}\, dw$$

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

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