STAT 400
UIUC

Answers for 7.1 (Part 3)

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1. A machine makes ½-inch ball bearings. In a random sample of 41 bearings, the sample standard deviation of the diameters of the bearings was 0.02 inch. Assume that the diameters of the bearings are approximately normally distributed. Construct a 90% confidence interval for the standard deviation of the diameters of the bearings.

s = 0.02. n = 41.

The confidence interval : $\left(\sqrt{\dfrac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}}}, \sqrt{\dfrac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}}} \right)$.

90% confidence level $\alpha = 0.10$ $\alpha/2 = \mathbf{0.05}$.

$n - 1 =$ **40** degrees of freedom.

$\chi^2_{\alpha/2} = \chi^2_{0.05} = \mathbf{55.76}$. $\chi^2_{1-\alpha/2} = \chi^2_{0.95} = \mathbf{26.51}$.

$$\left(\sqrt{\frac{(41-1)\cdot 0.02^2}{55.76}}, \sqrt{\frac{(41-1)\cdot 0.02^2}{26.51}} \right)$$

(0.01694 , 0.02457)

2. The following random sample was obtained from $N(\mu, \sigma^2)$ distribution:

16 12 18 13 21 15 8 17

Recall: $\bar{x} = 15$, $s^2 = 16$, $s = 4$.

a) Construct a 95% confidence interval for the overall standard deviation.

Confidence Interval for σ^2 : $\left(\dfrac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}}, \dfrac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}} \right)$.

$\alpha = 0.05.$ $\qquad \alpha/2 = 0.025.$ $\qquad 1 - \alpha/2 = 0.975.$

number of degrees of freedom = $n - 1 = 8 - 1 = 7.$

$\chi^2_{\alpha/2} = 16.01.$ $\qquad \chi^2_{1-\alpha/2} = 1.690.$

$$\left(\frac{(8-1)\cdot 16}{16.01}, \frac{(8-1)\cdot 16}{1.690} \right) \qquad (6.9956 ; 66.2722)$$

Confidence Interval for σ : $\left(\sqrt{6.9956}, \sqrt{66.2722} \right) =$ **(2.645 ; 8.141)**

b) Construct a 95% confidence lower bound for the overall standard deviation.

$$\left(\sqrt{\frac{(n-1)\cdot s^2}{\chi^2_{\alpha}}}, \infty \right)$$

7 degrees of freedom $\qquad \chi^2_{0.05} = 14.07.$

$$\left(\sqrt{\frac{(8-1)\cdot 16}{14.07}}, \infty \right)$$

(2.82 ; ∞)

c) Construct a 95% confidence upper bound for the overall standard deviation.

95% conf. upper bound for σ: $\left(0, \sqrt{\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha}}} \right) = \left(0, \sqrt{\frac{(8-1)\cdot 16}{2.167}} \right) =$ **(0, 7.19)**