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# Answers for 7.2, 8.2

**1.** Dr. Statman claims that his new revolutionary study method "Study While You Sleep" is more effective than the traditional study methods. In an experiment, 250 students enrolled in the same section of STAT 100 at UIUC were divided into two groups. One hundred students volunteered to study using SWYS method, and the other 150 students did whatever students usually do. At the end of the semester, the averages of the total number of points (out of 500) were compared for the two groups.

Note: This is **NOT** a good experiment design!

| | SWYS | Traditional |
|---|---|---|
| (sample) average total points | 450 | 410 |
| (sample) standard deviation | 20 | 45 |

a) Construct a 95% confidence interval for the difference in the average total points for SWYS and traditional study methods.

$n_1 = 100$ and $n_2 = 150$ are large, $t_{\alpha/2}$ can be approximated by $z_{\alpha/2}$.

$$\left(\overline{X} - \overline{Y}\right) \pm z_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad (450 - 410) \pm 1.96 \cdot \sqrt{\frac{20^2}{100} + \frac{45^2}{150}}$$

**$40 \pm 8.2$** **$(31.8, 48.2)$**

b) Perform the appropriate test at a 1% level of significance.

Claim: $\mu_{\text{New}} > \mu_{\text{Old}}$ $\quad H_0: \mu_{\text{New}} - \mu_{\text{Old}} = 0$ vs. $H_1: \mu_{\text{New}} - \mu_{\text{Old}} > 0$

Test Statistic: $$Z = \frac{\left(\overline{X} - \overline{Y}\right) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(450 - 410) - 0}{\sqrt{\frac{20^2}{100} + \frac{45^2}{150}}} = 9.562.$$

$n_1$ and $n_2$ are large

Rejection Region: Reject $H_0$ if $Z > z_{0.01} = 2.326$.

**Reject $H_0$** at $\alpha = 0.01$.

c) Test $H_0: \mu_S - \mu_T \le 30$ vs. $H_1: \mu_S - \mu_T > 30$ at $\alpha = 0.05$.

Test Statistic:
$$Z = \frac{(\overline{X} - \overline{Y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(450 - 410) - 30}{\sqrt{\frac{20^2}{100} + \frac{45^2}{150}}} = 2.39.$$

Rejection Region: Reject $H_0$ if $Z > z_{0.05} = 1.645$.

**Reject $H_0$** at $\alpha = 0.05$ p-value $\approx 0.0084$.

**2.** Two work designs are being considered for possible adoption in an assembly plant. A time study is conducted with 10 workers using design A and 12 workers using design B. The sample means and sample standard deviations of their assembly times (in minutes) are

| | Design A | Design B |
|---|---|---|
| Sample Mean | 78.3 | 85.6 |
| Sample Standard deviation | 4.8 | 6.5 |

Construct a 90% confidence interval for the difference in the mean assembly times between design A and Design B. Use Welch's *T*.

$$\left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\cdot\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\cdot\left(\frac{s_2^2}{n_2}\right)^2} \right\rfloor = \left\lfloor \frac{\left(\frac{4.8^2}{10} + \frac{6.5^2}{12}\right)^2}{\frac{1}{10 - 1}\cdot\left(\frac{4.8^2}{10}\right)^2 + \frac{1}{12 - 1}\cdot\left(\frac{6.5^2}{12}\right)^2} \right\rfloor$$

$= \lfloor 19.76324 \rfloor = 19$ degrees of freedom

$$(\overline{X} - \overline{Y}) \pm t_{\alpha/2}\cdot\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad (78.3 - 85.6) \pm 1.729\cdot\sqrt{\frac{4.8^2}{10} + \frac{6.5^2}{12}}$$

$\mathbf{-7.3 \pm 4.173}$ $\mathbf{(-11.473, -3.127)}$

**3.** A national equal employment opportunities committee is conducting an investigation to determine if women employees are as well paid as their male counterparts in comparable jobs. Random samples of 14 males and 11 females in junior academic positions are selected, and the following calculations are obtained from their salary data.

| | Male | Female |
|---|---|---|
| Sample Mean | \$48,530 | \$47,620 |
| Sample Standard deviation | 780 | 750 |

Assume that the populations are normally distributed with equal variances.

a) Construct a 95% confidence interval for the difference between the mean salaries of males and females in junior academic positions.

$$s^2_{\text{pooled}} = \frac{(14-1)\cdot 780^2 + (11-1)\cdot 750^2}{14+11-2} = 588{,}443.47826$$

$$s_{\text{pooled}} = 767.1$$

$$\left(\overline{\mathrm{X}} - \overline{\mathrm{Y}}\right) \pm t_{\alpha/2} \cdot s_{\text{pooled}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

14 + 11 – 2 = 23 degrees of freedom

$$(48{,}530 - 47{,}620) \pm 2.069 \cdot 767.1 \cdot \sqrt{\frac{1}{14} + \frac{1}{11}}$$

**910 ± 639.47** **( 270.53 , 1,549.47 )**

b) What is the p-value of the test $\mathrm{H}_0 : \mu_{\text{Male}} = \mu_{\text{Female}}$ vs. $\mathrm{H}_1 : \mu_{\text{Male}} \neq \mu_{\text{Female}}$?

Test Statistic:

$$\mathrm{T} = \frac{\left(\overline{\mathrm{X}} - \overline{\mathrm{Y}}\right) - \delta_0}{\mathrm{s}_{\text{pooled}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(48{,}530 - 47{,}620) - 0}{767.1 \cdot \sqrt{\frac{1}{14} + \frac{1}{11}}} = 2.944.$$

$$t_{0.005}(n_1 + n_2 - 2 = 23 \text{ d.f.}) = 2.807.$$

p-value ( 2 – tailed ) < 0.005 × 2 = 0.01 ( p-value ≈ 0.0073 )

**4.** A new revolutionary diet-and-exercise plan is introduced. Eight participants were weighed in the beginning of the program, and then again a week later. The results were as follows:

| Participant | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Weight Before | 213 | 222 | 232 | 201 | 230 | 188 | 218 | 182 |
| Weight After | 207 | 220 | 224 | 198 | 219 | 183 | 220 | 175 |
| Pounds Lost | 6 | 2 | 8 | 3 | 11 | 5 | $-2$ | 7 |

a) Construct a 90% confidence interval for the average number of pounds lost during one week on that plan.

$$\overline{\text{D}} = \frac{\sum d_i}{n} = \frac{6+2+8+3+11+5-2+7}{8} = \frac{40}{8} = 5 \text{ pounds.}$$

| $d$ | $d^2$ |
|---|---|
| 6 | 36 |
| 2 | 4 |
| 8 | 64 |
| 3 | 9 |
| 11 | 121 |
| 5 | 25 |
| $-2$ | 4 |
| 7 | 49 |
| | 312 |

OR

| $d$ | $d-\overline{\text{D}}$ | $(d-\overline{\text{D}})^2$ |
|---|---|---|
| 6 | 1 | 1 |
| 2 | $-3$ | 9 |
| 8 | 3 | 9 |
| 3 | $-2$ | 4 |
| 11 | 6 | 36 |
| 5 | 0 | 0 |
| $-2$ | $-7$ | 49 |
| 7 | 2 | 4 |
| | 0 | 112 |

$$s_{\text{D}}^2 = \frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1} = \frac{312 - \frac{(40)^2}{8}}{7} = 16. \qquad s_{\text{D}}^2 = \frac{\sum (d-\overline{\text{D}})^2}{n-1} = \frac{112}{7} = 16.$$

$$s_{\text{D}} = \sqrt{s_{\text{D}}^2} = \sqrt{16} = 4 \text{ pounds.}$$

Confidence interval: $\overline{\text{D}} \pm t_{\alpha/2} \cdot \frac{s_{\text{D}}}{\sqrt{n}}$. $n-1 = 7$ degrees of freedom.

90% confidence level, $\alpha = 0.10$, $\alpha/2 = 0.05$, $t_{\alpha/2} = 1.895$.

$5 \pm 1.895 \cdot \frac{4}{\sqrt{8}}$ **$5 \pm 2.68$** **$(2.32, 7.68)$**

b) Is there enough evidence to conclude that the average weight loss is less than 7 pounds per week? (Use $\alpha = 0.05$.) What is the p-value of this test?

Claim: $\delta < 7$ $H_0 : \delta \geq 7$ $H_1 : \delta < 7$

Test Statistic: $$T = \frac{\overline{D} - \delta_0}{s_D / \sqrt{n}} = \frac{5 - 7}{4 / \sqrt{8}} = -1.4142.$$

Rejection Region: Reject $H_0$ if $T < -t_{0.05}(8 - 1 = 7 \text{ d.f.}) = -1.895.$

The value of the test statistic **does not** fall into the Rejection Region.

**Do NOT Reject $H_0$ at $\alpha = 0.05$**. (p-value $\approx$ 0.10.)