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Examples for 7.3, 8.3 (Part 2)

Consider two dichotomous populations, with "success" proportions p_1 and p_2, respectively. Consider the sample proportions

$$\hat{p}_1 = \frac{x_1}{n_1} \quad \text{and} \quad \hat{p}_2 = \frac{x_2}{n_2}$$

where n_1 and n_2 are the sample sizes and x_1 and x_2 are the numbers of "successes" in the two samples from populations 1 and 2, respectively.

If n_1 and n_2 are large, then $(\hat{p}_1 - \hat{p}_2)$ is approximately normal with mean $p_1 - p_2$ and standard deviation $\sqrt{\frac{p_1 \cdot (1 - p_1)}{n_1} + \frac{p_2 \cdot (1 - p_2)}{n_2}}$.

The confidence interval for the difference between two population proportions $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1 \cdot (1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 \cdot (1 - \hat{p}_2)}{n_2}}$$

$H_0: p_1 = p_2$

$H_1: p_1 < p_2$

$H_1: p_1 > p_2$

$H_1: p_1 \neq p_2$

Test Statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p} \cdot (1 - \hat{p}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}, \qquad \text{where } \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 \cdot \hat{p}_1 + n_2 \cdot \hat{p}_2}{n_1 + n_2}.$$

1. In a comparative study of two new drugs, A and B, 120 patients were treated with drug A and 150 patients with drug B, and the following results were obtained.

	Drug A	Drug B
Cured	78	111
Not cured	42	39
Total	120	150

a) Construct a 95% confidence interval for the difference in the cure rates of the two drugs.

b) We wish to test whether drug B has a higher cure rate than drug A. Find the p-value of the appropriate test.