# Answers for 7.3, 8.3 (Part 2)

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Consider two dichotomous populations, with "success" proportions $p_1$ and $p_2$, respectively. Consider the sample proportions

$$\hat{p}_1 = \frac{x_1}{n_1} \quad \text{and} \quad \hat{p}_2 = \frac{x_2}{n_2}$$

where $n_1$ and $n_2$ are the sample sizes and $x_1$ and $x_2$ are the numbers of "successes" in the two samples from populations 1 and 2, respectively.

If $n_1$ and $n_2$ are large, then $(\hat{p}_1 - \hat{p}_2)$ is approximately normal with mean $p_1 - p_2$ and standard deviation $\sqrt{\frac{p_1 \cdot (1-p_1)}{n_1} + \frac{p_2 \cdot (1-p_2)}{n_2}}$.

The confidence interval for the difference between two population proportions $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1 \cdot (1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2 \cdot (1-\hat{p}_2)}{n_2}}$$

$H_0: p_1 = p_2$

$H_1: p_1 < p_2$

$H_1: p_1 > p_2$

$H_1: p_1 \neq p_2$

Test Statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p} \cdot (1-\hat{p}) \cdot \left( 1/n_1 + 1/n_2 \right)}}, \qquad \text{where } \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 \cdot \hat{p}_1 + n_2 \cdot \hat{p}_2}{n_1 + n_2}.$$

**1.** In a comparative study of two new drugs, A and B, 120 patients were treated with drug A and 150 patients with drug B, and the following results were obtained.

| | Drug A | Drug B |
|---|---|---|
| Cured | 78 | 111 |
| Not cured | 42 | 39 |
| Total | 120 | 150 |

a) Construct a 95% confidence interval for the difference in the cure rates of the two drugs.

$$\hat{p}_A = \frac{78}{120} = 0.65 \qquad \hat{p}_B = \frac{111}{150} = 0.74$$

95% confidence level $\alpha = 0.05$ $\alpha/2 = 0.025.$ $z_{\alpha/2} = 1.960.$

$$(0.74 - 0.65) \pm 1.96 \cdot \sqrt{\frac{0.74 \cdot (1 - 0.74)}{150} + \frac{0.65 \cdot (1 - 0.65)}{120}}$$

$\mathbf{0.09 \pm 0.11}$ $\mathbf{(-0.02, 0.20)}$

b) We wish to test whether drug B has a higher cure rate than drug A. Find the p-value of the appropriate test.

Claim: $p_A < p_B$

$H_0: p_A = p_B$ vs. $H_1: p_A < p_B$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{78 + 111}{120 + 150} = \frac{189}{270} = 0.70.$$

Test Statistic: $$Z = \frac{0.65 - 0.74}{\sqrt{0.70 \cdot 0.30 \cdot \left(\frac{1}{120} + \frac{1}{150}\right)}} = \mathbf{-1.60}.$$

P-value: Left – tailed.

P-value = P( Z < – 1.60 ) = **0.0548**.