

The  $\chi^2$  Distribution

$r$	$\chi^2_{0.995}$	$\chi^2_{0.99}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.90}$	$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.01}$	$\chi^2_{0.005}$
24	9.886	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
40	20.71	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69	66.77

1.  $s = 0.02$ .  $n = 41$ .

a) Claim:  $\sigma^2 \leq 0.0003$

$$H_0 : \sigma^2 \leq 0.0003 \quad \text{vs.} \quad H_1 : \sigma^2 > 0.0003$$

$s = 0.02$ .  $n = 41$ .

$$\text{Test Statistic: } \chi^2 = \frac{(n-1) \cdot s^2}{\sigma_0^2} = \frac{(41-1) \cdot 0.02^2}{0.0003} = \mathbf{53.3333}.$$

Rejection Region: Right – tailed.

Reject  $H_0$  if  $\chi^2 > \chi^2_{\alpha}$   $n - 1 = 40$  degrees of freedom.

$$\alpha = 0.05 \quad \chi^2_{0.05} = \mathbf{55.76}.$$

Reject  $H_0$  if  $\chi^2 > 55.76$ .

The value of the test statistic **does not** fall into the Rejection Region.

**Do NOT Reject  $H_0$  at  $\alpha = 0.05$ .**

b)  $\alpha = 0.10$   $\chi^2_{0.10} = \mathbf{51.80}$ .

Reject  $H_0$  if  $\chi^2 > 51.80$ .

The value of the test statistic **does** fall into the Rejection Region.

**Reject  $H_0$  at  $\alpha = 0.10$ .**

2. Claim:  $\sigma < 0.5$

$$H_0 : \sigma \geq 0.5 \quad \text{vs.} \quad H_1 : \sigma < 0.5$$

$$s = 0.4. \quad n = 25.$$

$$\text{Test Statistic:} \quad \chi^2 = \frac{(n-1) \cdot s^2}{\sigma_0^2} = \frac{(25-1) \cdot 0.4^2}{0.5^2} = \mathbf{15.36}.$$

Rejection Region: Left – tailed.

Reject  $H_0$  if  $\chi^2 < \chi_{1-\alpha}^2$   $n - 1 = 24$  degrees of freedom.

$$\alpha = 0.05 \quad \chi_{0.95}^2 = \mathbf{13.85}.$$

Reject  $H_0$  if  $\chi^2 < 13.85$ .

The value of the test statistic **does not** fall into the Rejection Region.

**Do NOT Reject  $H_0$  at  $\alpha = 0.05$ .**

3. Claim:  $\sigma^2 \leq 12$

$$H_0 : \sigma^2 \leq 12 \quad \text{vs.} \quad H_1 : \sigma^2 > 12$$

$$s^2 = 16. \quad n = 25.$$

$$\text{Test Statistic:} \quad \chi^2 = \frac{(n-1) \cdot s^2}{\sigma_0^2} = \frac{(25-1) \cdot 16}{12} = \mathbf{32}.$$

Rejection Region: Right – tailed.

Reject  $H_0$  if  $\chi^2 > \chi_{\alpha}^2$   $n - 1 = 24$  degrees of freedom.

$$\alpha = 0.05 \quad \chi_{0.05}^2 = \mathbf{36.42}.$$

Reject  $H_0$  if  $\chi^2 > 36.42$ .

The value of the test statistic **does not** fall into the Rejection Region.

**Do NOT Reject  $H_0$  at  $\alpha = 0.05$ .**