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Answers for 8.3, 8.1

1. A scientist wishes to test if a new treatment has a better cure rate than the traditional treatment which cures only 60% of the patients. In order to test whether the new treatment is more effective or not, a test group of 20 patients were given the new treatment. Assume that each personal result is independent of the others.

Trying to decide: cure rate $p \leq 0.60$ vs. $p > 0.60$.

a) If the new treatment has the same success rate as the traditional, what is the probability that at least 14 out of 20 patients (14 or more) will be cured?

$P(X \geq 14 \mid p = 0.60) = 1 - CDF(13 \mid p = 0.60) = 1 - 0.750 = \mathbf{0.250}$.

b) Suppose that 14 out of 20 patients in the test group were cured. Based on the answer for part (a), is there a reason to believe that the new treatment has a better cure rate than the traditional treatment?

If $p = 0.60$, then 25% of all possible samples would have 14 or more patients cured (out of 20). Thus, it is not unusual to see 14 out of 20 patients cured for a treatment that cures 60% of the patients. We have no reason to believe that the new treatment has a better cure rate than the traditional treatment if $X = 14$.

c) If the new treatment has the same success rate as the traditional, what is the probability that at least 17 out of 20 patients (17 or more) will be cured?

$P(X \geq 17 \mid p = 0.60) = 1 - CDF(16 \mid p = 0.60) = 1 - 0.984 = \mathbf{0.016}$.

d) Suppose that 17 out of 20 patients in the test group were cured. Based on the answer for part (c), is there a reason to believe that the new treatment has a better cure rate than the traditional treatment?

If $p = 0.60$, then only 1.6% of all possible samples would have 17 or more patients cured (out of 20). Thus, it is fairly unusual to see 17 out of 20 patients cured for a treatment that cures 60% of the patients. We have a good reason to believe that the new treatment has a better cure rate than the traditional treatment if $X = 17$.

2. A certain automobile manufacturer claims that at least 80% of its cars meet the tough new standards of the Environmental Protection Agency (EPA). Let p denote the proportion of the cars that meet the new EPA standards. The EPA tests a random sample of 400 its cars, suppose that 308 of the 400 cars in our sample meet the new EPA standards.

a) Perform an appropriate test at a 10% level of significance ($\alpha = 0.10$).

Claim: $p \geq 0.80$ $\quad$ $H_0: p \geq 0.80$ vs. $H_1: p < 0.80$

Y = 308. $\quad$ n = 400. $\quad$ $\hat{p} = \frac{Y}{n} = \frac{308}{400} = 0.77$.

Test Statistic: $$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1 - p_0)}{n}}} = \frac{0.77 - 0.80}{\sqrt{\frac{0.80 \cdot 0.20}{400}}} = \mathbf{-1.50}.$$

Rejection Region: Left – tailed. $\quad$ Reject H_0 if $Z < -z_{\alpha}$

$\alpha = 0.10$ $\quad$ $z_{0.10} = 1.282$. $\quad$ Reject H_0 if $Z < -1.282$.

Decision: The value of the test statistic DOES fall into the Rejection Region.

Reject H_0 at $\alpha = 0.10$.

b) Perform an appropriate test at a 5% level of significance ($\alpha = 0.05$).

Rejection Region: Left – tailed. $\quad$ Reject H_0 if $Z < -z_{\alpha}$

$\alpha = 0.05$ $\quad$ $z_{0.05} = 1.645$. $\quad$ Reject H_0 if $Z < -1.645$.

Decision: The value of the test statistic does NOT fall into the Rejection Region.

Do NOT Reject H_0 at $\alpha = 0.05$.

c) Find the p-value of the appropriate test.

Left – tailed. $\quad$ P-value = $P(Z \leq -1.50)$ = **0.0668**.

d) Using the p-value from part (c), state your decision (Accept H_0 or Reject H_0) at $\alpha = 0.08$.

0.0668 = p-value < $\alpha = 0.08$. $\quad$ **Reject H_0 at $\alpha = 0.08$.**

3. Alex wants to test whether a coin is fair or not. Suppose he observes 477 heads in 900 tosses. Let p denote the probability of obtaining heads.

a) Perform the appropriate test using a 10% level of significance.

Claim: $p = 0.50$

$H_0: p = 0.50$ vs. $H_1: p \neq 0.50$

Y = 477. $n = 900$. $\hat{p} = \frac{Y}{n} = \frac{477}{900} = 0.53.$

Test Statistic: $$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1 - p_0)}{n}}} = \frac{0.53 - 0.50}{\sqrt{\frac{0.50 \cdot 0.50}{900}}} = \mathbf{1.80}.$$

Rejection Region: Two – tailed.

Reject H_0 if $Z < -z_{\alpha/2}$ or $Z > z_{\alpha/2}$

$\alpha = 0.10$ $\alpha/2 = \mathbf{0.05}$. $z_{0.05} = \mathbf{1.645}$.

Reject H_0 if $Z < -1.645$ or $Z > 1.645$.

Decision:

The value of the test statistic **does** fall into the Rejection Region.

Reject H_0 at $\alpha = 0.10$.

OR

P-value: Two – tailed.

P-value = P(| Z | > 1.80) = 2 · 0.0359 = **0.0718**.

Decision:

0.0718 = p-value < α = 0.10. **Reject H_0 at $\alpha = 0.10$.**

b) Find the p-value of the test in part (a).

Two – tailed. P-value = P(| Z | > 1.80) = 2 · 0.0359 = **0.0718**.

c) Using the p-value from part (b), state your decision (Accept H_0 or Reject H_0) for $\alpha = 0.05$.

0.0718 = p-value > α = 0.05. **Do NOT Reject H_0 at $\alpha = 0.05$.**

4. $H_0: p \le 0.20$ vs. $H_1: p > 0.20$.

Y = 72. $n = 300$.

Compute the p-value.

State your decision at $\alpha = 0.05$.

$$\hat{p} = \frac{Y}{n} = \frac{72}{300} = 0.24.$$

Test Statistic: $$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1 - p_0)}{n}}} = \frac{0.24 - 0.20}{\sqrt{\frac{0.20 \cdot 0.80}{300}}} = \mathbf{1.73}.$$

P-value: Rightt – tailed. P-value = P(Z > 1.73) = **0.0418**.

0.0418 = p-value < α = 0.05. **Reject H_0 at α = 0.05**.