STAT 400 Fall 2017

Interval and Testing Practice

1. In a random sample of 100 *Hawk & Hummingbird Airline* (*HHA*) direct flights from New York to Boston, the average number of passengers was 56.3, with sample standard deviation 11.33.

a) Construct a 95% confidence interval for the overall average number of passengers on this route.

b) To make a reasonable profit, *HHA* must average at least 58 passengers per flight on this route. The president of the airline is concerned that the average number of passengers is less than 58. Perform the appropriate test at $\alpha = 0.05$.

c) Suppose the actual overall average number of passengers on this flight is 57. Then in part (b) a (Type I Error, Type II Error, correct decision) was made.

d) Find the p-value of the test in part (b).

e) Using the P-value obtained in part (d), state your decision (Accept H_0 or Reject H_0) for $\alpha = 0.10$.

f) Describe in the context of the problem what happens if Type I Error occurs.

g) Describe in the context of the problem what happens if Type II Error occurs.

2. *Hawk & Hummingbird Airline* wants to determine the proportion of passengers that bring only carry-on luggage to the flight from New York to Boston.

a) In a random sample of 200 passengers, 44 passengers had only carry-on luggage. Is there enough evidence to conclude that more than 20% of all passengers have only carry-on luggage? Use $\alpha = 0.10$.

b) Find the p-value of the test in part (a).

c) Find a 90% confidence interval for the overall proportion of passengers who have only carry-on luggage.

d) Find the minimum sample size required in order to estimate the proportion of passengers who have only carry-on luggage to within 2% with 90% confidence, if it is known that this proportion is at most 0.30.

3. In a random sample of 25 direct flights from New York to Boston by *Hawk & Hummingbird Airline*, the sample mean flight time was 56 minutes and the sample standard deviation was 8 minutes. (Assume the flight times are approximately normally distributed.)

a) Construct a 98% confidence interval for the overall mean flight time on this route.

b) Test whether the actual overall mean flight time on this route is 1 hour (60 minutes) or it is different at 5% significance level.

c) Find the P-value of the test in part (b).

d) Using the P-value obtained in part (c), state your decision (Accept H_0 or Reject H_0) for $\alpha = 0.01$.

e) Suppose that the actual overall mean flight time on this route is 58 minutes. Then in part (b) a (Type I Error, Type II Error, correct decision) was made.

4. The manager of a department store is thinking about establishing a new billing system for the store's credit customers. After a thorough financial analysis, she determines that the new system will be cost effective only if the mean monthly account is more than \$170. A random sample of 169 monthly accounts is drawn for which the sample mean is \$177 and the sample standard deviation is \$65.

a) Can the manager conclude from this that the new system will be cost effective? Perform the appropriate test using a 5% level of significance.

b) Suppose that the true value of the overall mean monthly account is \$175. Then in part (a), (Type I Error, Type II Error, correct decision) was made.

c) What is the p-value of the test in part (a) ?

d) Using the p-value from part (c), state your decision (Accept H_0 or Reject H_0) for $\alpha = 0.10$.

e) Describe in the context of the problem what happens when a Type I Error occurs.

f) Describe in the context of the problem what happens when a Type II Error occurs.

g) Construct a 95% confidence interval for the overall mean monthly account.

h) How large should the sample be if the manager wants to estimate the overall mean monthly account to within \$5 with 95% confidence? (Use the sample standard deviation as an estimate of the population standard deviation.)

5. Suppose that the lengths of the trout fry in a pond at the fish hatchery have the overall standard deviation of 0.8 inch. A random sample of 49 fry will be netted and their lengths measured.

a) What is the probability that the sample mean will be within 0.2 inch of the population mean μ?

b) Suppose the sample mean of the 49 fry netted is $\bar{x} = 3.4$ inches. Construct a 95% confidence interval for the overall mean lengths of the trout fry in the pond.

c) What is the minimum sample size required for estimating the overall mean lengths of the trout fry in the pond to within 0.2 inch with 95% confidence?

d) Construct a 90% confidence interval for the overall mean lengths of the trout fry in the pond.

e) Construct a 94% confidence interval for the overall mean lengths of the trout fry in the pond.

6. Mercury makes a 2.4 liter V-6 engine. The company's engineers believe that the standard deviation of power delivered by the engine is 8 HP.

a) A potential buyer wants to estimate the average power and intends to sample 100 engines (each engine to be run a single time). What is the probability that the sample mean will differ from the average power delivered by the engine by more than 2 HP?

b) How many engines need to be tested if we want to estimate the average power to within 2 HP with 90% confidence?

7. A manufacturer of video games wants to install some machines in shopping malls. In a pilot study on the potential profitability of this enterprise, games were placed for one week in 10 randomly chosen shopping malls. The weekly profits in dollars were as follows:

110.80	67.90	141.20	93.60	75.80
131.30	106.40	87.80	94.10	98.00

Assume the population distribution is normal. Construct a 90% confidence interval for the mean weekly profit for these games in all shopping malls.

[Hint: $\Sigma x = 1{,}006.90$, $\Sigma x^2 = 106{,}059.39$.]

8. In a random sample of 20 homeowners in Anytown, the average monthly electric bill during June was \$64. The sample standard deviation was \$11. Assume electric bills are approximately normally distributed. The manager of the Anytown Power Plant believes that the variance of June monthly electric bills is 100 (\$ squared). Test whether the variance of June monthly electric bills is 100 or it is different at a 10% level of significance.

9. Several employees of *Bob's Computer Warehouse* copmlain that the variance of the amounts in employees' salary is too large. Bob wants to check whether the variance in employees' monthly salaries is greater than 10,000 (\$ squared). A random sample of size 31 employees give a variance of 13,000. Perform the appropriate test at $\alpha = 0.05$ level of significance.

10. The proportion of defective items is not supposed to be over 15%. A buyer wants to test whether the proportion of defectives exceeds the allowable limit. The buyer takes a random sample of 100 items and finds that 19 are defective.

a) Construct a 95% confidence interval for the overall proportion of defective items.

b) What is the minimum sample size required in order to estimate the overall proportion of defective items to within 3% with 95% confidence? (Assume that the overall proportion of defective items is at most 0.20.)

c) Perform the appropriate test at $\alpha = 0.05$.

d) Find the p-value of the test in part (c).

e) Using the p-value from part (d), state your decision (Accept H_0 or Reject H_0) at $\alpha = 0.10$.

11 – 12. A leakage test was conducted to determine the effectiveness of a seal designed To keep the inside of a plug airtight. An air needle was inserted into the plug, and the plug and needle were placed under water. The pressure was then increased until leakage was observed. Let X equal the pressure in pounds per square inch. Assume that X follows a normal distribution. The following 10 observations of X were recorded:

3.1	3.3	4.5	2.8	3.5	3.5	3.7	4.2	3.9	3.3

11. a) Find a point estimate of μ using the observations.

b) Find a point estimate of σ using the observations.

c) Find a 95% confidence upper bound for μ.

12. d) Construct a 95% (two-sided) confidence interval for σ.

e) Find a 95% confidence upper bound for σ.

Answers:

1. $\overline{X} = 56.3$, $s = 11.33$, $n = 100$. (n is large, can use z instead of t)

a) $\mathbf{56.3 \pm 2.22}$ or $\mathbf{(54.08\,,\,58.52)}$ ($z_{0.025} = 1.960$)

b) $H_0 : \mu \geq 58$ vs. $H_1 : \mu < 58$. Test Statistic: $Z = \mathbf{-1.50}$.

Rejection Region: $Z < -z_{0.05} = \mathbf{-1.645}$. **Accept H_0.**

c) **Type II Error**.

d) p-value = **0.0668**.

e) **Reject H_0.**

f) The route is profitable, but the airline cancels it.

g) The route is not profitable, but the airline keeps it.

2. $n = 200$, $X = 44$, $\hat{p} = \mathbf{0.22}$.

a) $H_0 : p \leq 0.20$ vs. $H_1 : p > 0.20$. Test Statistic: $Z = \mathbf{0.71}$.

Rejection Region: $Z > z_{0.10} = \mathbf{1.282}$. **Accept H_0.**

b) p-value = **0.2389**.

c) $\mathbf{0.22 \pm 0.048}$ or $\mathbf{(0.172\,,\,0.268)}$ ($z_{0.05} = 1.645$)

d) $n = \mathbf{1421}$ (use $p = 0.30$)

3. $\overline{X} = 56$, $s = 8$, $n = 25$.

a) $\mathbf{56 \pm 4}$ or $\mathbf{(52\,,\,60)}$ ($t_{0.01} = 2.492$, 24 degrees of freedom)

b) $H_0 : \mu = 60$ vs. $H_1 : \mu \neq 60$. Test Statistic: $T = \mathbf{-2.50}$.

Rejection Region: $T < -t_{0.025} = \mathbf{-2.064}$ or $T > t_{0.025} = \mathbf{2.064}$.

(24 degrees of freedom) **Reject H_0.**

c) **0.01** < p-value < **0.02** (closer to 0.02). p-value = one tail × 2.

d) **Accept H_0.**

e) **Correct decision**.

4. $\overline{X} = 177$, $s = 65$, $n = 169$.

a) $H_0 : \mu \le 170$ vs. $H_1 : \mu > 170$. Test Statistic: $Z = \mathbf{1.40}$.

Rejection Region: $Z > z_{0.05} = \mathbf{1.645}$. **Accept H_0.**

b) **Type II Error**.

c) p-value = **0.0808**.

d) **Reject H_0.**

e) The new system is not cost effective, but it is adopted.

f) The new system is cost effective, but it is not adopted.

g) **177 ± 9.8** or **(167.2 , 186.8)** $(z_{0.025} = 1.960)$

h) $n = \mathbf{650}$.

5. $\mu = ?$, $\sigma = 0.8$, $n = 49$.

a) Central Limit Theorem

$P(| \overline{X} - \mu | < 0.2) = P(| Z | < 1.75) = \mathbf{0.9198}$.

b) **3.4 ± 0.224**. c) $n = \mathbf{62}$.

d) **3.4 ± 0.188**. e) **3.4 ± 0.215**.

6. $\mu = ?$, $\sigma = 8$.

a) $n = 100$. Central Limit Theorem

$P(| \overline{X} - \mu | > 2) = P(| Z | > 2.50) = \mathbf{0.0124}$.

b) $n = \mathbf{44}$.

7. $\overline{X} = 100.69$, $s^2 = 519.4$, $s = 22.79$.

100.69 ± 13.21 $(t_{0.05} = 1.833,\ 9 \text{ d.f.})$.

8. $s = 11, \quad n = 20.$

$H_0 : \sigma^2 = 100$ vs. $H_1 : \sigma^2 \neq 100.$ Test Statistic: $\chi^2 = \mathbf{22.99}$.

Rejection Region: $\chi^2 < \chi^2_{0.95} = \mathbf{10.1170}$ or $\chi^2 > \chi^2_{0.05} = \mathbf{30.1435}$.

(19 degrees of freedom) **Accept H_0.**

9. $s^2 = 13{,}000, \quad n = 31.$

$H_0 : \sigma^2 \leq 10{,}000$ vs. $H_1 : \sigma^2 > 10{,}000.$ Test Statistic: $\chi^2 = \mathbf{39}$.

Rejection Region: $\chi^2 > \chi^2_{0.05} = \mathbf{43.77}$. (30 degrees of freedom)

Accept H_0.

10. $n = 100, \quad X = 19, \quad \hat{p} = \mathbf{0.19}$.

a) $\mathbf{0.19 \pm 0.077}$ or $\mathbf{(0.113, 0.267)}$ $(z_{0.025} = 1.960)$

b) $n = \mathbf{683}$ (use $p = 0.20$)

c) $H_0 : p \leq 0.15$ vs. $H_1 : p > 0.15.$ Test Statistic: $Z = \mathbf{1.12}$.

Rejection Region: $Z > z_{0.05} = \mathbf{1.645}$. **Accept H_0.**

d) p-value = **0.1314**.

e) **Accept H_0.**

11. **7.1-10** **6.2-12**

a) Find a point estimate of μ using the observations.

$\bar{x} = \mathbf{3.580}$;

b) Find a point estimate of σ using the observations.

$s^2 = \frac{2.356}{9} \approx 0.261778, \qquad s \approx \mathbf{0.51164}$;

c) Find a 95% confidence upper bound for μ.

$t_{0.05}(9) = 1.833,$

$\left(0,\ 3.580 + 1.833 \cdot 0.51164 / \sqrt{10}\right) = \mathbf{(0,\ 3.8766)}.$

12. d) Construct a 95% (two-sided) confidence interval for σ.

$\left(\sqrt{\frac{9 \cdot 0.261778}{19.02}}, \sqrt{\frac{9 \cdot 0.261778}{2.700}}\right) = \mathbf{(0.35195,\ 0.93413)}$;

e) Find a 95% confidence upper bound for σ.

$\left(0, \sqrt{\frac{9 \cdot 0.261778}{3.325}}\right) = \mathbf{(0,\ 0.84177)}$;